Speed Optimized Two-Qubit Gates with Laser Coherent Control Techniques for Ion Trap Quantum Computing

J. J. García-Ripoll,[1,*] P. Zoller,[2] and J. I. Cirac[1]

[1]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, Garching, D-85748, Germany*
[2]*Innsbruck Universität, Technikerstrasse 24, 6020 Innsbruck, Austria*

(Received 2 June 2003; published 7 October 2003)

We propose a new concept for a two-qubit gate operating on a pair of trapped ions based on laser coherent control techniques. The gate is insensitive to the temperature of the ions, works also outside the Lamb-Dicke regime, requires no individual addressing by lasers, and can be orders of magnitude faster than the trap period, which is presently the speed limit of all two-qubit proposals.

DOI: 10.1103/PhysRevLett.91.157901 PACS numbers: 03.67.Lx, 03.65.Vf, 03.67.Pp, 32.80.Qk

Trapped ions constitute one of the most promising systems to implement scalable quantum computation [1]. There, qubits are stored in long-lived internal atomic states and a universal set of gates is obtained by manipulating the internal states with lasers and entangling the ions via the motional states [2]. During the past years a remarkable experimental progress in building an ion trap quantum computer has allowed one to realize two-qubit gates [3–6] and to prepare entangled states [7–9]. The ultimate challenge now is the development of scalable ion trap quantum computing, where ions are stored and moved to different regions to perform the required gates [10,11]. Basic steps towards this goal have already been demonstrated experimentally [12].

An important question is to identify the current limitations of the two-qubit gates with trapped ions (given the fact that one-qubit gates are significantly simpler with those systems). The ideal scheme should (i) be independent of temperature (so that one does not need to cool the ions to their ground state after they are moved to or from their storage area); (ii) require no addressability (to allow the ions to be as close as possible during the gate so as to strengthen their interaction), and (iii) be fast (in order to minimize the effects of decoherence during the gate, and to speed up the computation). This last property has been identified [1] as a key limitation: in essentially all schemes suggested so far [2,13–17] one has to resolve spectroscopically the motional sidebands of the ions with the exciting laser, which limits the laser intensity and therefore the gate time.

The two-qubit gate between pairs of ions analyzed below solves the problem of speed by using mechanical effects instead of spectral methods to couple the motion and internal states of the ions. In this way the new limits on the time of the quantum gate are those of laser control, which can be orders of magnitude faster than the present limits dictated by trap design. This implies a significant step forward towards fast and efficient scalable quantum computations with trapped ions.

We will first study the dynamics of two ions under the influence of short laser pulses with varying directions. We will prove that there exist certain laser pulse sequences which perform a phase gate on the two qubits, while leaving the motional state unchanged. We illustrate this with two protocols for laser pulses: (i) a sequence of four pulses which gives a gate time of $T = 1.08/\nu$ with ν the trap frequency, and (ii) a protocol which allows us to perform a gate in a time $T \sim N_P^{-2/3}/\nu$ where N_P is the number of laser pulses. Finally, we will complement our study of the gate dynamics with an analysis of possible errors, which includes fluctuations of the intensity or the duration of the pulses, and temperature. The gate will be shown to be extremely robust to these perturbations.

We consider two ions in a one-dimensional harmonic trap, interacting with a laser beam on resonance. The Hamiltonian describing this situation [2] can be written as $H = H_0 + H_1$, where $H_0 = \nu_c a^\dagger a + \nu_r b^\dagger b$ describes the motion in the trap and

$$H_1 = \frac{\Omega(t)}{2}\sigma_1^+ e^{i\eta_c(a^\dagger + a) + i/2\eta_r(b^\dagger + b)} + \frac{\Omega(t)}{2}\sigma_2^+ e^{i\eta_c(a^\dagger + a) - i/2\eta_r(b^\dagger + b)} + \text{H.c.} \quad (1)$$

Here, $\nu_c = \nu$ and $\nu_r = \sqrt{3}\nu_c$ are the frequencies of the center of mass and stretching mode, respectively; a and b are the corresponding annihilation operators, and $\eta_c = \eta/\sqrt{2}$ and $\eta_r = \eta\sqrt[4]{4/3}$ are proportional to the Lamb-Dicke parameter, η. The Rabi frequency Ω is the same for both ions, since we have not assumed individual addressing. Also notice that replacing η with $(-\eta)$ is equivalent to reversing the direction of the laser beam.

In the following we will consider two different kind of processes: (i) free evolution, where the laser is switched off ($\Omega = 0$) for a certain time; (ii) sequences of pairs of very fast laser pulses, each of them coming from opposite sides. If we denote by δt the duration of a pulse and by Ω the corresponding Rabi frequency, we are interested in the limit $\delta t \to 0$ with $\Omega\, \delta t = \pi$. Processes (i) and (ii) will be alternated [see Fig. 1(a)]: at time t_1 a sequence of z_1 pulses is applied, followed by free evolution until at time t_2 another sequence of z_2 pulses is applied followed

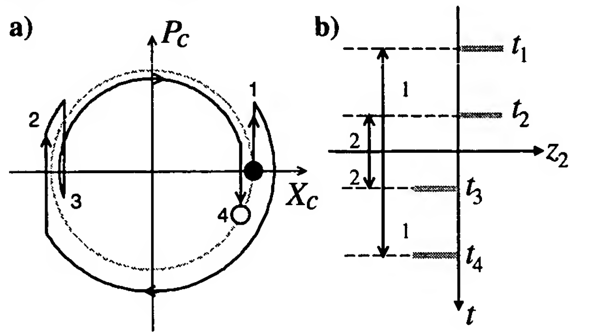

FIG. 1. (a) Trajectory in phase space of the center-of-mass state (X_c, P_c) [where $(X_c + iP_c)/\sqrt{2} = \langle a \rangle$] during the two-qubit gate (solid line), connecting the initial state (black circle) to the final state (grey circle) at gate time T. The time evolution consists of a sequence of kicks (vertical displacements), which are interspersed with free harmonic oscillator evolution (motion along the arcs). A pulse sequence satisfying the commensurability condition (3) guarantees that the final phase space point is restored to the one corresponding to a free harmonic evolution (dashed circle). The particular pulse sequence plotted corresponds to a four pulse sequence given in the text (protocol I). (b) shows how the laser pulses (bars) distribute in time for this scheme.

by free evolution and so on. The z_k are integer numbers, whose sign indicates the direction of the laser pulses.

For a pulse sequence, consisting of kicks interspersed with free harmonic time evolution (Fig. 1), we write the evolution operator as $\mathcal{U} = \mathcal{U}_c \mathcal{U}_r$, where $\mathcal{U}_{c,r} = \prod_{k=1}^N U_{c,r}(\Delta t_k, z_k)$ has contributions of the center of mass and relative motions,

$$U_c(t_k, z_k) = e^{-i2z_k\eta_c(a+a^\dagger)(\sigma_1^z+\sigma_2^z)}e^{-i\nu_c\Delta t_k a^\dagger a},$$
$$U_r(t_k, z_k) = e^{-iz_k\eta_r(b+b^\dagger)(\sigma_1^z-\sigma_2^z)}e^{-i\nu_r\Delta t_k b^\dagger b}.$$

The integers z_k indicate the direction of the initial pulse in the sequence of pairs of very fast laser pulses, each of them coming from opposite sites.

In order to fully characterize $\mathcal{U}$, we have only to investigate its action on states of the form $|i\rangle_1|j\rangle_2|\alpha\rangle_c|\beta\rangle_r$, where $i, j = 0, 1$ denote the computational basis, and $|\alpha\rangle$ and $|\beta\rangle$ are coherent states. This task can be easily carried out once we know the action of $\mathcal{U} = \prod_{k=1}^N U(\phi_k, p_k)$ on an arbitrary coherent state $|\alpha\rangle$, where

$$U(\phi_k, p_k) = e^{-ip(a+a^\dagger)}e^{-i\phi_k a^\dagger a}.$$

We obtain $\mathcal{U}|\alpha\rangle = e^{i\xi}|\tilde{\alpha}\rangle$, with $\theta_k = \sum_{m=1}^k \phi_m$, and where

$$\tilde{\alpha} = \alpha e^{-i\theta_N} - i\sum_{k=1}^N p_k e^{i(\theta_k-\theta_N)},$$

$$\xi = -\sum_{m=2}^N\sum_{k=1}^{m-1} p_m p_k \sin(\theta_k - \theta_m) - \Re\left[\alpha\sum_{k=0}^N p_k e^{-i\theta_m}\right].$$

The crucial point is to realize that if $\sum_{k=1}^N p_k e^{i\theta_k} = 0$ the motional state $|\alpha\rangle$ after the evolution is the same as if there were only free evolution [Fig. 1(a)], and a global phase ξ appears which does not depend on the motional state. We obtain the conditions

$$C_c \equiv \sum_{k=1}^N z_k e^{-i\nu t_k} = 0, \qquad C_r \equiv \sum_{k=1}^N z_k e^{-i\sqrt{3}\nu t_k} = 0. \quad (3)$$

If these commensurability conditions are satisfied, the motional state of the ion will not depend on the qubits and the evolution operator will be given by

$$\mathcal{U}(\Theta) = e^{i\Theta\sigma_1^z\sigma_2^z}e^{-i\nu_c T a^\dagger a}e^{-i\nu_r T b^\dagger b}. \quad (4)$$

The value T is the total time required by the gate and

$$\Theta = 4\eta^2\sum_{m=2}^N\sum_{k=1}^{m-1} z_k z_m\left[\frac{\sin[\sqrt{3}\nu\Delta t_{km}]}{\sqrt{3}} - \sin(\nu\Delta t_{km})\right], \quad (5)$$

where $\Delta t_{km} = t_k - t_m$. Therefore, if Eqs. (3) are fulfilled, and $\Theta = \pi/4$ we will have a controlled-phase gate which is completely independent of the initial motional state, i.e., there are no temperature requirements.

It is straightforward to show that for any value of T it is always possible to find a sequence of laser pulses which implements the gate, and therefore the gate operation can be, in principle, arbitrarily fast. The search for this sequence may be done numerically, or even semianalytically. In the following we give two simple (not optimized) protocols.

The first protocol (protocol I) requires the least number of pulses to produce the gate in a fixed time $T \simeq 1.08(2\pi/\nu)$. The recipe is illustrated in Fig. 1, which provides the phase space plots for the evolution of the motional state. The sequence of pulses is defined as $(z_n/N, t_n) = \{(\gamma, -\tau_1), (1, -\tau_2), (-1, \tau_2), (-\gamma, \tau_1)\}$. Here $0 < \gamma = \cos(\theta) < 1.0$ is a real number, which may be introduced by tilting both lasers a small angle θ with respect to the trap axis. It is always possible to find a solution to Eq. (3) with $\tau_1 \simeq 0.538(4)(2\pi/\nu) > \tau_2 > 0$. The results are summarized in Fig. 2. As shown in

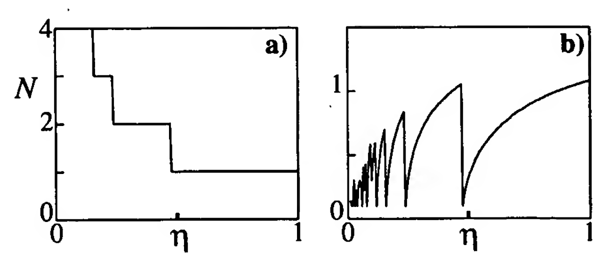

FIG. 2. (a) Number of pairs of pulses and (b) relative angle of the two laser beams required to produce a phase gate using the first exact scheme developed in the paper.

Fig. 2(a), for realistic values of the Lamb-Dicke parameter [4] we need only to apply the sequence of pulses 1 or 2 times to implement a phase gate.

The second protocol (protocol II) performs the gate in an arbitrarily short time T. The pulses are distributed according to $(z_n/N, t_n) = \{(-2, -\tau_1), (3, -\tau_2), (-2, -\tau_3), (2, \tau_3), (-3, \tau_2), (2, \tau_1)\}$. The process requires $N_p = \sum |z_n| = 14N$ pairs of pulses and takes $T = 2\tau_1$. As Fig. 2 shows, the number of pulses increases with decreasing time as $N_p \propto T^{-3/2}$.

In order to study the main potential limitations, we define the error of the gate E in terms of the gate fidelity [18] as $E = 1 - \mathrm{Tr}_{\mathrm{mot}}\{\mathcal{Q}_{\mathrm{mot}}\rho_{\mathrm{mot}}\mathcal{Q}_{\mathrm{mot}}^\dagger\}$. Here $\mathrm{Tr}_{\mathrm{mot}}$ and $\mathrm{Tr}_{\mathrm{int}}$ denote traces over motional and internal degrees of freedom, and $\mathcal{Q}_{\mathrm{mot}} = \mathrm{Tr}_{\mathrm{int}}\{\mathcal{U}(\pi/4)U_{\mathrm{real}}^\dagger\}$ depends on U_{real}, the gate performed in the presence of imperfections.

We now turn to a discussion of the possible sources of errors. A limiting factor for the gate is the anharmonicities of the restoring forces. The more pulses we apply, the larger the relative displacement of the ions, as Fig. 3(b) shows. When the ions become too close to each other, the increasing intensity of the Coulomb force can lead to a breakdown of the harmonic approximation which is implicit in Eq. (1). In order to analyze this effect, we have made a perturbative analysis of the anharmonic corrections induced by the Coulomb force and found that for $\nu T \ll 1$ they cause an error $E \simeq |0.4a_0/d|^2/(2\pi\nu T)$, where a_0 is the ground state size of the external potential and d is the ion separation in equilibrium. For typical parameters and imposing an error $E \simeq 10^{-4}$ we obtain $\nu T \simeq 10^{-3}$. Similar results are obtained when applied to the anharmonicities of the trap itself.

We have also studied the influence of errors in the laser pulses of our scheme. Up to now, our analytical calculations assumed that one may neglect the influence of the trap during the laser pulses. To validate this assumption we have simulated numerically a system of two ions with only one vibrational mode. We have used the exact sequences to produce the phase gate using only eight laser pulses. In Fig. 4(a) we plot the error as a function of the pulse duration, $\tau = \pi/2\Omega$. The longer the pulse, the more important the effect of the trap, and the larger the error. But even for relatively long pulses, we obtain a fidelity which is comparable to the results obtained in current setups [4–6]. We have also studied the influence of intensity noise, or, equivalently, random errors in the pulse duration. The larger the amplitude of the error the lower the fidelity of the gate, as Fig. 4(b) shows.

As mentioned before, the scheme is insensitive to temperature. If the commensurability condition (3) is not perfectly satisfied due to, for example, errors in the timing of the laser pulses, or misalignment of the lasers, then the corresponding contribution to the gate error is $E = (C_1^4 + C_2^4 + 4C_1C_2 - 6)/8$, with

$$C_1 = \exp[-(1/2 + k_bT/\hbar\nu_c)|2\eta_cC_c|^2],$$
$$C_2 = \exp[-(1/2 + k_bT/\hbar\nu_r)|\eta_rC_r|^2],$$

which is a smooth function of temperature T.

Finally, we include some remarks regarding the experimental implementation. First, it is not necessary to kick the atoms using pairs of counterpropagating laser beams. The same effect may also be achieved in current experiments by reverting the internal state of both ions simultaneously. One needs only a laser aligned with the trap to kick the atoms and another laser orthogonal to the axis of the trap to produce the NOT gate. The second and more important remark is that it is possible to avoid errors in the laser pulses by using more sophisticated kicking methods. One possibility consists in using stimulated Raman adiabatic passage (STIRAP) [19,20]. Only one of the qubit states would be connected by two on-resonance laser beams to a third atomic state, $|e'\rangle$. In the first part of the kicking process, the Rabi frequencies Ω_a and Ω_b are adiabatically switched on and off, respectively. The momenta of both laser beams should be different, so that as we slowly proceed from $\Omega_a/\Omega_b \simeq 0$ to the opposite regime $\Omega_b/\Omega_a \simeq 0$, the ions in the state $|1\rangle$ are

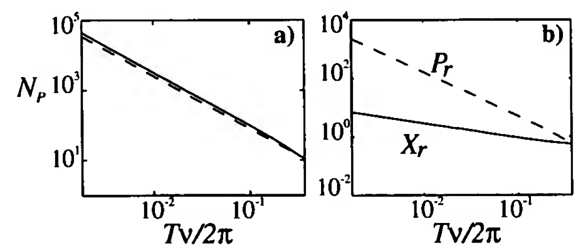

FIG. 3. (a) Log-log plot of the number of pairs of pulses using protocol II, as a function of T, for $\eta = 0.178$ [4] [exact result (solid line) and a rough estimate $N_P = 40(\nu T/2\pi)^{-3/2}$ (dashed line) based on perturbative calculations]. (b) Maximum relative displacement, X_r (solid line), and maximum momentum acquired, P_r (dashed line), for scheme II, with, $X_r = \max[\langle x_r(t)\rangle/a_0]$, and $P_r \max[\langle p_r(t)\rangle a_0/\hbar]$.

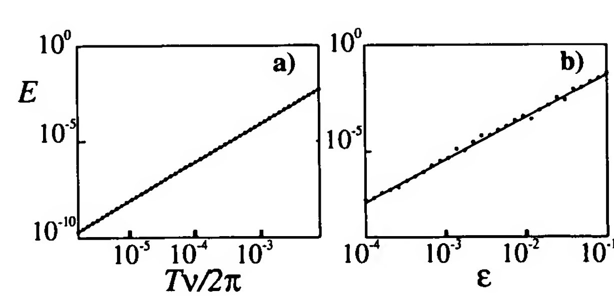

FIG. 4. Four kicks sequence for $\eta = 0.178$ and $T = \mathcal{O}(1/\nu)$: (a) Error vs $\nu\tau$. It perfectly fits the estimate $E = 2(\tau\nu/2\pi)^2$. (b) Mean error for random errors in the duration $\tau_k = \pi/(2\Omega)(1 + \epsilon r_k)$, with random numbers r_k uniformly distributed in $[-1/2, 1/2]$ (solid line corresponds to$E = 4\epsilon^2$.

completely transferred to the new dark state $|e'\rangle$ and get a kick $|1\rangle \rightarrow e^{i(\vec{k}_a-\vec{k}_b)\vec{x}}|e'\rangle$. Next we must change the sense of the laser beams ($\vec{k}_{a,b} \rightarrow -\vec{k}_{a,b}$) and perform the adiabatic transfer from $\Omega_b/\Omega_a \simeq 0$ to $\Omega_a/\Omega_b \simeq 0$. The advantages of this method are (i) the system remains all the time in a dark state, avoiding spontaneous emission; (ii) the process is insensitive to fluctuations of the intensity; (iii) the duration of the pulse need not be precisely adjusted, and (iv) the intensity of the laser need not be the same for both ions. Finally, one should also mention that for short pulses the laser bandwidth may get broad, so that in order to avoid problems with the hyperfine atomic structure one may use atomic species with no nuclear spin.

Summing up, in this work we have developed a new concept of two-qubit quantum gate for trapped ions, in which the trap frequency no longer poses a limitation on the speed of the gate. The limitations in that case come from (i) the anharmonicities of the restoring force that the ions experience when pushed far away from each other, and (ii) the ability to control the laser pulses. The first limitation still allows one to perform the gates in a time which is 3 orders of magnitude smaller than the one imposed by the trap frequency. The second limitation can be overcome by using adiabatic passage techniques. In addition, our scheme is independent of the temperature, requires no addressability, and works beyond the Lamb-Dicke regime. In any case, the rapid experimental progress in laser control with very short pulses indicates that it may soon be possible to perform quantum gates with a very high speed.

J. I. C. thanks D. Leibfried for discussions and NIST for the hospitality during his stay in Boulder. P. Z. thanks the Innsbruck ion trap group for discussions. This work was supported by the EU IST projects RESQ and TOPQIP, the DFG (Schwerpunktprogramm Quanteninformationsverarbeitung), the Kompetenznetzwerk Quanteninformationsverarbeitung der Bayerischen Staatsregierung, the Austrian Science Foundation, and the Institute for Quantum Information.

*Electronic address: Juan.Ripoll@mpq.mpg.de

[1] B. G. Levi, Phys. Today **56**, No. 5, 17–19 (2003).
[2] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).
[3] C. Monroe *et al.*, Phys. Rev. Lett. **75**, 4714 (1995).
[4] B. DeMarco *et al.*, Phys. Rev. Lett. **89**, 267901 (2002).
[5] D. Leibfried *et al.*, Nature (London) **422**, 412 (2003).
[6] F. Schmidt-Kaler *et al.*, Nature (London) **422**, 408 (2003).
[7] Q. A. Turchette *et al.*, Phys. Rev. Lett. **81**, 3631 (1998).
[8] V. Meyer *et al.*, Phys. Rev. Lett. **86**, 5870 (2001).
[9] C. A. Sackett *et al.*, Nature (London) **404**, 256 (2000).
[10] J. I. Cirac and P. Zoller, Nature (London) **404**, 579 (2000).
[11] D. Kielpinksi, C. Monroe, and D. J. Wineland, Nature (London) **417**, 709 (2002).
[12] M. A. Rowe *et al.*, Quantum Inf. Comput. **2**, 257 (2002).
[13] A. Sørensen and K. Mølmer, Phys. Rev. A **62**, 022311 (2000).
[14] A. Sørensen and K. Mølmer, Phys. Rev. Lett. **82**, 1971–1974 (1999).
[15] D. Jonathan, M. B. Plenio, and P. L. Knight, Phys. Rev. A **62**, 042307 (2000).
[16] G. J. Milburn, S. Schneider, and D. F. V. James, Fortschr. Phys. **48**, 801–810 (2000).
[17] A notable exception is J. F. Poyatos., J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **81**, 1322 (1998). However, here highly nonharmonic traps are required and still the gate time is limited by the trap frequency.
[18] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[19] M. Weitz, B. C. Young, and S. Chu, Phys. Rev. A **50**, 2438 (1993).
[20] K. Bergmann, H. Theuer, and B. W. Shore, Rev. Mod. Phys. **70**, 1003 (1998).